Estimating Ocean Surface Level using the Intrinsic Non-stationary Covariance Function

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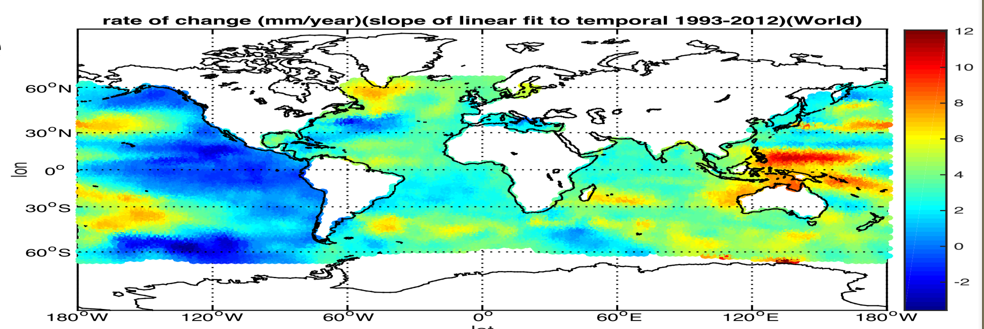
NG33A-1851

Motivation

The aim of this work is to characterize the spatio-temporal scales of variability from the sea-level datasets in order to aid in the understanding of different emerging patterns of regional sea-level changes.



Model

$$f(u,t) = g(t) + l(u)(t - t_0) + m(u,t)$$

where: f ~ sea level at *u* (latitude, longitude) and *t* (time),
g ~ global mean sea level at *t*,
l ~ local mean sea level trend at *u*,
m ~ emergence of forced signals at *u* and *t*.

Proposed Solution

$$m(u,t) \sim GP(0, K(u,t))$$

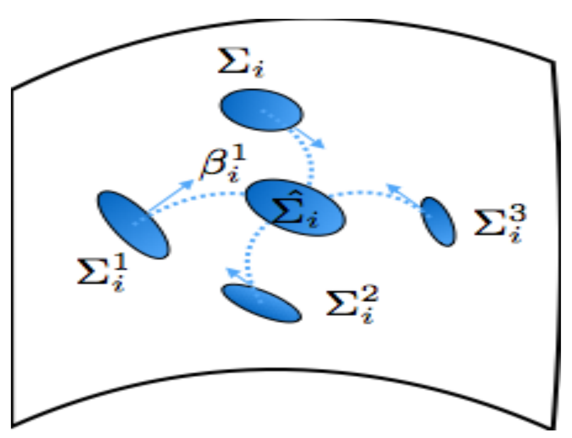
Characterize the emerging pattern using the Gaussian process regression (GP) with a zero mean and a full covariance matrix (K) that captures the underlying random field.

Proposed solution for designing the covariance matrix that captures the variable local scale ($\Sigma(u_i, t_i)$).

Model the variable correlation structure of the underlying random field for (Σ_i) using the intrinsic statistics on a Riemannian manifold.

Intrinsic Mean: $\hat{\Sigma}_i := argmin_{\hat{\Sigma}_i}(\frac{1}{N}\sum_{k=1}^{N} D^2(\Sigma_i^k, \hat{\Sigma}_i))$

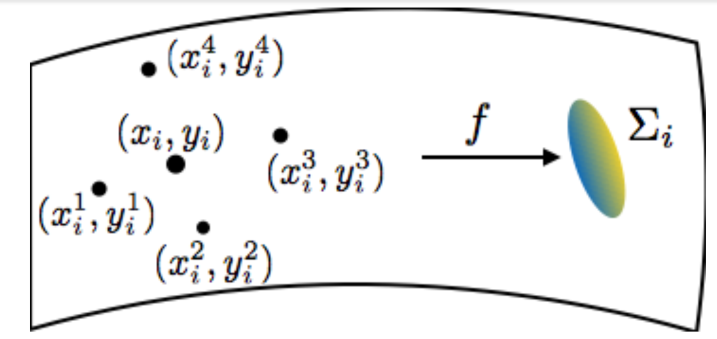
Riemannian Metric: $D^2(\Sigma_i, \Sigma_j)$



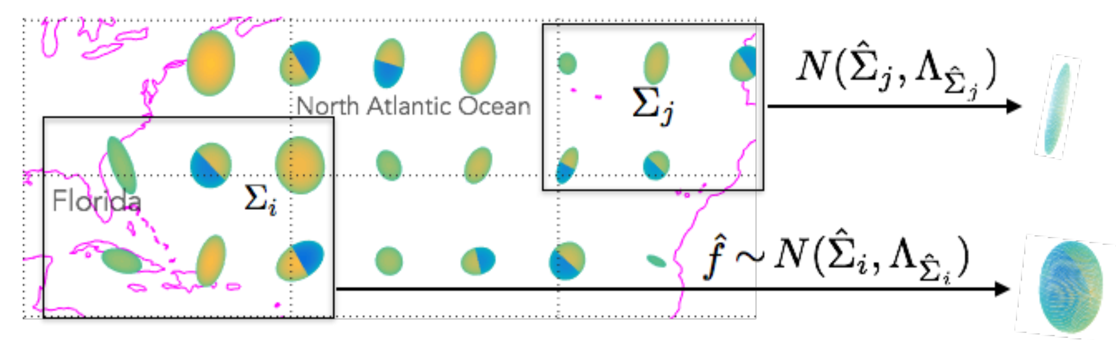
Method

Use the satellite altimeter datasets $\{x_i$ (latitude, longitude, time), y_i (ocean surface level (mm)$\}$

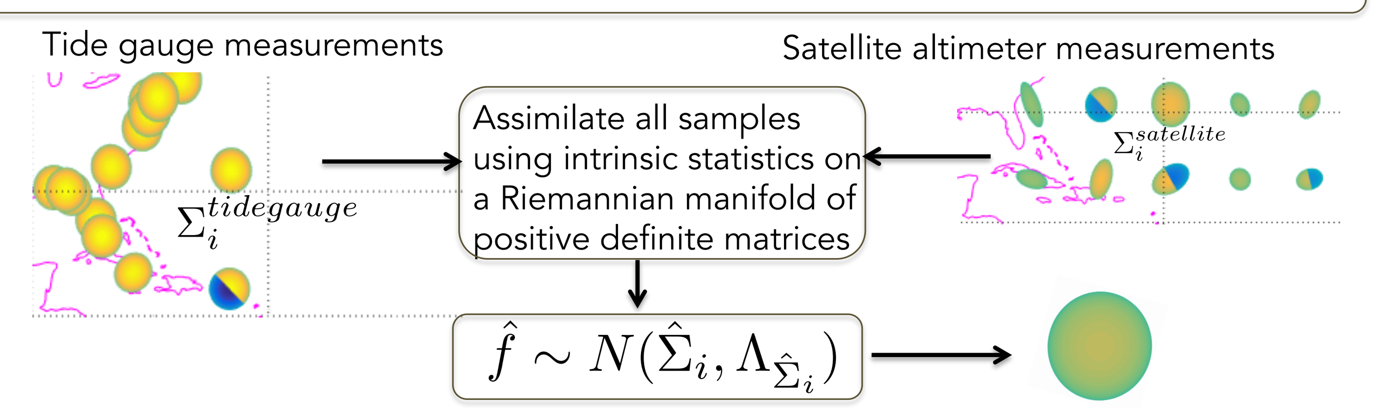
Initialize kernels (scale matrix) using the local Gaussian process $\{\Sigma_i(x_i)\}$



Estimate the intrinsic non-stationary covariance function $\{\hat{\Sigma}_i(\Sigma_i)\}$



Estimate the intrinsic data fusion non-stationary covariance function $\{\hat{\Sigma}_i(\Sigma_i^S, \Sigma_i^T)\}$



Estimate the final estimates at every test point (x_i^*, y_i^*) using the global Gaussian process (GP)

Results

Evaluation of Spatio-temporal and Spatial Datasets

Methods (Units of mm/year)	Metric	Spatial-temporal satellite dataset (with data assimilation from tide gauge measurements)	Spatial tide gauge dataset (with data fusion from spatial satellite dataset)
Stationary Gaussian Process (GP)	MSE	1.38	0.85
	nLPD	4.29	2.81
Non-stationary GP	MSE	1.18	0.75
	nLPD	4.25	2.82
Intrinsic Non-stationary GP	MSE	1.10	0.71
	nLPD	4.15	2.78
Intrinsic Data Fusion Non-stationary GP	MSE	**1.09**	**0.66**
	nLPD	**4.11**	**2.75**

Conclusion

The improvement in the evaluation metrics using our proposed covariance function, as seen in the above table of the satellite altimeter and tide gauge datasets, suggests improvements in the statistical estimates of the spatio-temporal ocean surface level.

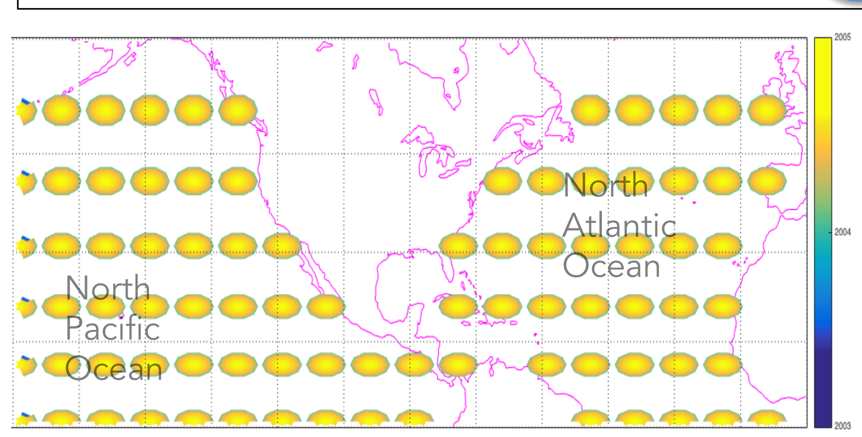
Key advantages of the proposed covariance function:

- Modeling the non-stationarity of the underlying stochastic process at a global scale
- Non-parametrically modeling the boundaries of the regional geophysical variability

Background

I) Stationary Covariance Function

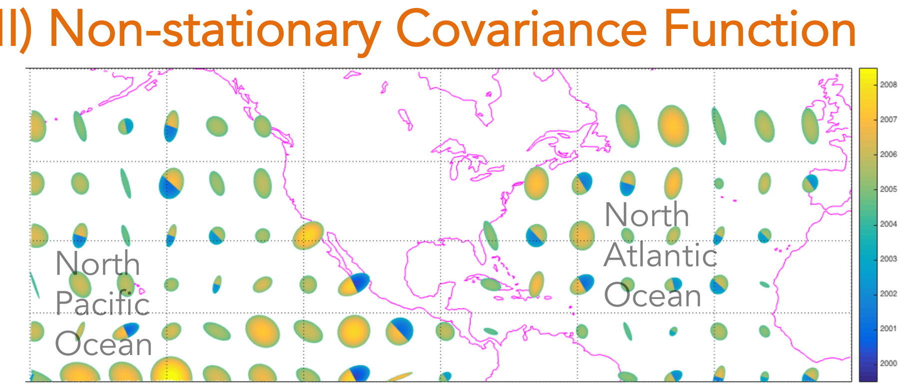
Northeast America Coastline — Local Scale of Variability (Σ_i)



- Scale of variability in space and time is the same everywhere.

$$Cov_{stationary}(x_i, x_j) \propto exp(-(x_i - x_j)^T \Sigma^{-1}(x_i - x_j))$$

II) Non-stationary Covariance Function

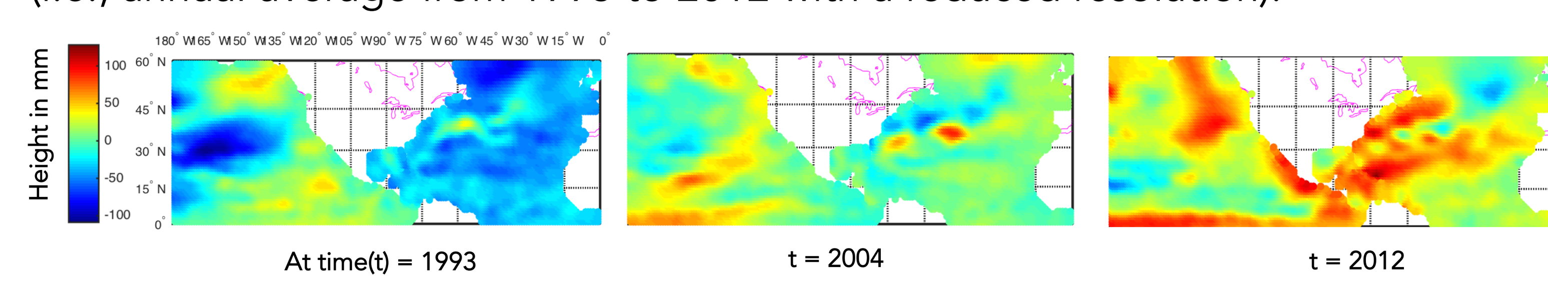


- Scale of variability depends on the neighborhood correlation structure.

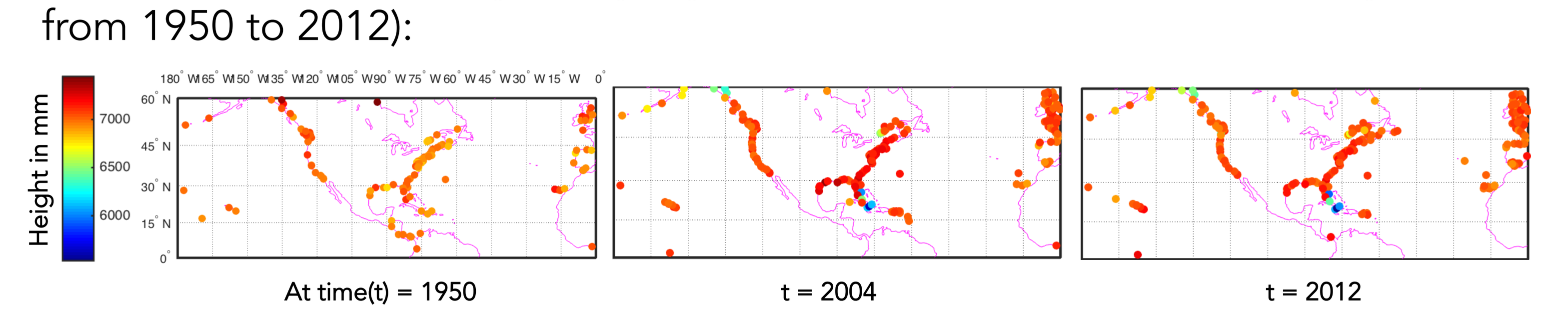
$$Cov_{Non-Stat}(x_i, x_j) \propto exp(-(x_i - x_j)^T (\frac{\Sigma_i + \Sigma_j}{2})^{-1}(x_i - x_j))$$

Datasets

- Estimate the ocean surface level using satellite altimeter measurements (i.e., annual average from 1993 to 2012 with a reduced resolution):



- Assimilate the data using tide gauge measurements (i.e., annual average from 1950 to 2012):



Evaluation Metrics

- Standardized Mean Square Error: $MSE = \frac{1}{n}\sum_{i=1}^{n}\frac{(y_i - y_i^*)^2}{\text{var}(y)}$
- Negative Log Predictive Density: $nLPD = -\frac{1}{n}\sum_{i=1}^{n}\log(p(y_i|x_i))$.

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Acknowledgments

This work is supported by the U.S Department of Homeland Security (CCICADA) under Grant # 2012-ST-104-000044.

The author, Chintan Dalal, would like to thank Doug Nychka and Dimitris Metaxas for their valuable feedback.

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